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The Breaking of Subnuclear Democracy as the Origin of Flavour Mixing

HARALD FRITZSCH*+

Theory Division, CERN, CH-1211 Geneva 23, Switzerland

and

Max–Planck–Institut für Physik und Astrophysik
Werner–Heisenberg–Institut für Physik, Munich

and

DIRK HOLTMANNSPÖTTER

Sektion Physik, Universität München

Abstract

It is shown that the simplest breaking of the subnuclear democracy leads to a successful description of the mixing between the second and third family. In the lepton channel the $\nu_\mu - \nu_\tau$ oscillations are expected to be described by a mixing angle of $2.65°$ which might be observed soon in neutrino experiments.

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In the standard electroweak model both the masses of the quarks as well as the weak mixing angles enter as free parameters. Any further insight into the yet unknown dynamics of mass generation would imply a step beyond the physics of the electroweak standard model. At present it seems far too early to attempt an actual solution of the dynamics of mass generation, and one is invited to follow a strategy similar to the one which led eventually to the solution of the strong interaction dynamics by QCD, by looking for specific patterns and symmetries as well as specific symmetry violations.

The mass spectra of the quarks are dominated strongly by the masses of the members of the third family, i. e. by t and b. Thus a clear hierarchical pattern exists. Furthermore the masses of the first family are small compared to those of the second one. Moreover, the CKM–mixing matrix exhibits a hierarchical pattern – the transitions between the second and third family as well as between the first and the third family are small compared to those between the first and the second family.

About 15 years ago, it was emphasized[1)] that the observed hierarchies signify that nature seems to be close to the so–called "rank–one" limit, in which all mixing angles vanish and both the u– and d–type mass matrices are proportional to the rank-one matrix

$$M_0 = \text{const.} \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{1}$$

Whether the dynamics of the mass generation allows that this limit can be achieved in a consistent way remains an unsolved issue. Encouraged by the observed hierarchical pattern of the masses and the mixing parameters, we shall assume that this is the case. In itself it is a non-trivial constraint and can be derived from imposing a chiral symmetry, as emphasized in ref. (2). This symmetry ensures that an electroweak doublet which is massless remains unmixed and is coupled to the W–boson with full strength. As soon as mass is introduced, at least for one member of the doublet, the symmetry is violated and mixing phenomena are expected to show up. That way a chiral evolution of the CKM matrix can be considered.[2)] At the first stage only the t and b quark masses are introduced, due to their non-vanishing coupling to the scalar "Higgs" field. The CKM–matrix is unity in this limit. At the next stage the second generation acquires a mass also. Since the (u, d)–doublet is still massless, only the second and the third generations mix, and the CKM–matrix is given by a real 2×2 rotation matrix in the $(c,\, s) - (t,\, b)$ subsystem, describing e. g. the mixing between s and b. Only at the next step, at which the u and d masses are introduced, does the full CKM–matrix appear, described in general by three angles and one phase.

It has been emphasized some time ago[3)] that the rank-one mass matrix (see eq. (1)) can be expressed in terms of a "democratic mass matrix":

$$M_0 = c \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} , \tag{2}$$

which exhibits an $S(3)_L \times S(3)_R$ symmetry. Writing down the mass eigenstates in terms of the eigenstates of the "democratic" symmetry, one finds e.g. for the lepton channel:

$$\begin{aligned} e^0 &= \frac{1}{\sqrt{2}}(l_1 - l_2) \\ \mu^0 &= \frac{1}{\sqrt{6}}(l_1 + l_2 - 2l_3) \\ \tau^0 &= \frac{1}{\sqrt{3}}(l_1 + l_2 + l_3) \end{aligned} \tag{3}$$

(l_i: symmetry eigenstates). Note that e^0 and μ^0 are massless in the limit considered here, and any linear combination of the first two state vectors given in eq. (3) would fulfil the same purpose, i. e. the decomposition is not unique, only the wave function of the coherent state τ^0 is uniquely defined. This ambiguity will disappear as soon as the symmetry is violated.

The wave functions given in eq. (3) are reminiscent of the wave functions of the neutral pseudoscalar mesons in QCD in the $SU(3)_L \times SU(3)_R$ limit:

$$\begin{aligned} \pi_0^0 &= \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) \qquad (4) \\ \eta_0 &= \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \\ \eta_0' &= \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s). \end{aligned}$$

(Here the lower index denotes that we are considering the chiral limit). Also the mass spectrum of these mesons is identical to the mass spectrum of the leptons and quarks in the "democratic" limit: two mesons (π_0^0, η_0) are massless and act as Nambu–Goldstone bosons, while the third coherent state η_0' is <u>not</u> massless due to the QCD anomaly.

In the chiral limit the (mass)^2–matrix of the neutral pseudoscalar mesons is also a "democratic" mass matrix when written in terms of the $(\bar{q}q)$– eigenstates $(\bar{u}u)$, $(\bar{d}d)$ and $(\bar{s}s)$[4]:

$$M^2(ps) = \lambda \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{5}$$

where the strength parameter λ is given by $\lambda = M^2(\eta'_0)\,/\,3$. The mass matrix (5) describes the result of the QCD–anomaly which causes strong transitions between the quark eigenstates (due to gluonic annihilation effects enhanced by topological effects). Likewise one may argue that analogous transitions are the reason for the lepton–quark mass hierarchy. Here we shall not speculate about a detailed mechanism of this type, but merely study the effect of symmetry breaking.

In the case of the pseudoscalar mesons the breaking of the symmetry down to $SU(2)_L \times SU(2)_R$ is provided by a direct mass term $m_s\bar{s}s$ for the s–quark. This implies a modification of the (3,3) matrix element in eq. (5), where λ is replaced by $\lambda + M^2(\bar{s}s)$ where $M^2(\bar{s}s)$ is given by $2M_k^2$, which is proportional to $< \bar{s}s >_0$, the expectation value of $\bar{s}s$ in the QCD vacuum. This direct mass term causes the violation of the symmetry and generates at the same time a mixing between η_0 and η'_0, a mass for the η_0, and a mass shift for the η'_0.

It would be interesting to see whether an analogue of the simplest violation of the "democratic" symmetry which describes successfully the mass and mixing pattern of the $\eta - \eta'$–system is also able to describe the observed mixing and mass pattern of the second and third family of leptons and quarks. Let us replace the (3,3) matrix element in eq. (2) by $1 + \varepsilon_i$; (i = l (lepton), u (u–quark), d (d–quark) respectively. The small real parameter ε describes the departure from democratic symmetry and leads

a) to a generation of mass for the second family and

b) to flavour mixing between the third and the second family. Since ε is directly related (see below) to a fermion mass and the latter is <u>not</u> restricted to be positive, ε can be positive or negative. (Note that a negative Fermi–Dirac mass can always be turned into a positive one by a suitable γ_5–transformation of the spin $\frac{1}{2}$ field). Since the original mass term is represented by a symmetric matrix, we take ε to be real.

First we study the mass and mixing pattern of the charged leptons. The mass operator (trace Θ^μ_μ of the energy–momentum tensor $\Theta_{\mu\nu}$) can be written as

$$\Theta^\mu_\mu = \Theta^{0\mu}_\mu + c_l \varepsilon_l \bar{l}_3 l_3 \tag{6}$$

where $\Theta^{0\mu}_\mu$ describes the mass term in the symmetry limit. The modification of the spectrum and the induced mixing can be obtained by considering the matrix elements:

$$\begin{aligned} < \mu^0 | c_l \varepsilon_l \bar{l}_3 l_3 | \mu^0 > &= +\frac{2}{3} c_l \varepsilon_l \\ < \tau^0 | c_l \varepsilon_l \bar{l}_3 l_3 | \tau^0 > &= +\frac{1}{3} c_l \varepsilon_l \\ < \mu^0 | c_l \varepsilon_l \bar{l}_3 l_3 | \tau^0 > &= -\frac{\sqrt{2}}{3} c_l \varepsilon_l \ . \end{aligned} \tag{7}$$

One observes that

a) the muon acquires a mass given by $c_l \cdot \varepsilon_l$ i. e. $m(\mu)/m(\tau) \cong \frac{2}{9}\varepsilon_l$;

b) the τ–lepton mass is changed slightly $(m(\tau)/m(\tau^0) \cong 1 + \frac{1}{9}\varepsilon_l)$;

c) the flavour mixing is induced by the fact that the perturbation proportional to $\bar{l}_3 l_3$ leads to a non–vanishing transition matrix element between μ^0 und τ^0.

This phenomenon is analogous to the chiral symmetry violation of QCD, where the s–quark mass term $m_s \bar{s} s$ leads to a mass for the η–meson, a mass shift for the η'–meson and a mixing between η and η'.

It is instructive to rewrite the mass matrix in the hierarchical basis, where one obtains, using the relations (7):

$$M = c_l \begin{pmatrix} 0 & 0 & 0 \\ 0 & +\frac{2}{3}\varepsilon_l & -\frac{\sqrt{2}}{3}\varepsilon_l \\ 0 & -\frac{\sqrt{2}}{3}\varepsilon_l & 3 + \frac{1}{3}\varepsilon_l \end{pmatrix} . \tag{8}$$

In lowest order of ε one finds the mass eigenvalues $m_\mu = \frac{2}{9}\varepsilon_l \cdot m_\tau\,, m_\tau = m_{\tau^0}\,, \Theta_{\mu\tau} = |\sqrt{2}\cdot\varepsilon_l/9|$.

The exact mass eigenvalues and the mixing angle are given by:

$$\begin{aligned}
m_1/c_l &= \frac{3+\varepsilon_l}{2} - \frac{3}{2}\sqrt{1-\frac{2}{9}\varepsilon_l+\frac{1}{9}\varepsilon_l^2} \\
m_2/c_l &= \frac{3+\varepsilon_l}{2} + \frac{3}{2}\sqrt{1-\frac{2}{9}\varepsilon_l+\frac{1}{9}\varepsilon_l^2} \\
\sin\Theta_l &= \frac{1}{\sqrt{2}}\left(1-\frac{1-\frac{1}{9}\varepsilon_l}{(1-\frac{2}{9}\varepsilon_l+\frac{1}{9}\varepsilon_l^2)^{1/2}}\right)^{1/2}
\end{aligned} \tag{9}$$

The ratio m_μ/m_τ, observed to be 0.0595, gives $\varepsilon_l = 0.286$ and a $\mu - \tau$ mixing angle of 2.65°. Whether this mixing angle is directly relevant for neutrino oscillations or not depends on the neutrino sector. For massless neutrinos the mixing angle does not have a direct physical meaning, i. e. it can be rotated away. If neutrinos have a mass, the neutrino mass matrix will in general induce further mixing angles. A general discussion would be beyond the scope of this paper.

However, we should like to consider an interesting scenario which is being discussed in connection with cosmological aspects. Let us suppose that the τ–neutrino mass is of the order of 10 eV in order to be relevant for the "missing matter problem" in cosmology, the muon neutrino is in the milli–eV range, i. e. $m(\nu_\mu) < 10^{-2}eV$, and the electron neutrino mass is neglected. The mass generation for the ν_μ–mass proceeds in an analogous way as discussed above for the muon mass. However, the ε–parameter for the neutrino sector is tiny ($< 5\cdot 10^{-2}$), and the mixing angle induced via the ν_μ–mass generation can safely be neglected. Thus the angle relevant for the $\nu_\mu - \nu_\tau$ oscillations remains 2.65°, i. e. $\sin^2 2\Theta = 0.0085$. This value is essentially the lowest limit given by the Charm II experiment[5)], i. e. is not ruled out for any value of $\triangle m^2 = m(\nu_\tau)^2 - m(\nu_\mu)^2$. However, the E531 experiment[6)] gives a limit of about $16eV^2$ for $\triangle m^2$, i. e. $m(\nu_\tau) < 4eV$. This limit seems to rule out a cosmological role with respect to the "missing matter" for the τ–neutrino. However, one might caution this conclusion since our mixing angle of 2.65° is not far from the limit of ($\sin^2 2\Theta = 0.004$), at which, according to the E531 experiment, all values of $m(\nu_\tau)$ are allowed. New experiments, e. g. the CHORUS and NOMAD experiments now or soon under way at CERN, will clarify this issue. If the mixing angle is 2.65° as argued above and the ν_τ–mass above 10 eV, one should observe the $\nu_\mu - \nu_\tau$– oscillations within one year[7)].

Replacing ε_l by ε_u, ε_d respectively, we can determine the symmetry breaking parameters for the quark sector. The ratio m_s/m_b is allowed to vary in the range $0.022 \ldots 0.044$ (see ref. (8)). According to eq. (9) one finds ε_u to vary from $\varepsilon_d = 0.11$ to 0.21. The associated $s - b$ mixing angle varies from $\Theta(s,b) = 1.0^\circ$ $(\sin\Theta = 0.018)$ and $\Theta(s,b) = 1.95^\circ$ $(\sin\Theta = 0.034)$. As an illustrative example we use the values $m_b(1GeV) = 5200MeV$, $m_s(1GeV) = 220MeV$. One obtains $\varepsilon_d = 0.20$ and $\sin\Theta(s,b) = 0.032$.

To determine the amount of mixing in the (c,t)–channel, a knowledge of the ratio m_c/m_t is required. As an illustrative example we take $m_c(1GeV) = 1.35GeV$, $m_t(1GeV) = 260GeV$ (i. e. $m_t(m_t) = 160GeV$), which gives $m_c/m_t \cong 0.005$. In this case one finds $\varepsilon_u = 0.023$ and $\Theta(c,t) = 0.21^\circ$ $(\sin\Theta(c,t) = 0.004)$.

The actual weak mixing between the third and the second quark family is combined effect of the two family mixings described above. The symmetry breaking given by the ε–parameter can be interpreted, as done in eq. (7), as a direct mass term for the $l_3(u_3, d_3)$ fermion system. However, a direct fermion mass term need not be positive, since its sign can always be changed by a suitable γ_5–transformation. What counts for our analysis is the relative sign of the m_s–mass term in comparison to the m_c–term, discussed previously. Thus two possibilities must be considered:

a) Both the m_s– and the m_c–term have the same relative sign with respect to each other, i. e. both ε_d and ε_u are positive, and the mixing angle between the second and third family is given by the difference $\Theta(sb) - \Theta(ct)$. This possibility seems to be ruled out by experiment, since it would lead to $V_{cb} < 0.03$.

b) The relative signs of the breaking terms ε_d and ε_u are different, and the mixing angle between the (s,b) and (c,t) systems is given by the sum $\Theta(sb) + \Theta(ct)$. Thus we obtain $V_{cb} \cong \sin(\Theta(sb) + \Theta(ct))$.

According to the range of values for m_s discussed above, one finds $V_{cb} \cong 0.022...0.038$. For example, for $m_s(1GeV) = 220MeV$, $m_c(1GeV) = 1.35GeV$, $m_t(1GeV) = 260GeV$ one finds $V_{cb} \cong 0.036$.

Before discussing the experimental situation, we add a comment about the mass generation for the first family, which at the same time will also generate the other mixing elements, e.g. V_{us} and V_{ub}, of the CKM matrix. These masses can be generated by a further breakdown of the symmetry, e. g. in the matrix of eq. (5) by a small departure of a second diagonal matrix element from unity. (This would correspond to a direct mass term for that state.) Due to the small values of the masses of the first family in comparison to the λ–scale, given by the mass of the third generation fermion (e.g. $m_e/\lambda = 0.0009$), the strength of this symmetry breaking is much smaller than the primary symmetry breaking, which leads to the masses for the second family. (The situation is analogous to the one in

hadronic physics, where the breaking of the chiral symmetry is primarily given by the mass of the s–quark, and the m_u/m_d mass terms can be neglected to a good approximation). In general it is expected, both from the arguments considered here and more generally from the analysis on chiral symmetry given in ref. (2), that the matrix elements V_{cb} and V_{ts} will be affected only by small corrections of order 10^{-3} or less in absolute magnitude (of order $\frac{m_d}{m_b}$, $\frac{m_u}{m_t}$ respectively). Thus the primary breaking of the democratic symmetry leads solely to a mixing between the second and the third family, and the secondary breaking, responsible for the Cabibbo angle etc., will not affect the 2×2 submatrix of the CKM–matrix describing the $s - b$ mixing in a significant way.

The experiments give $V_{cb} = 0.032 \ldots 0.054$[9)]. We conclude from the analysis given above that our ansatz for the symmetry breaking reproduces the lower part of the experimental range. According to a recent analysis the experimental data are reproduced best for $V_{cb} = 0.038 \pm 0.003$[10)], i. e. it seems that V_{cb} is lower than previously thought, consistent with our expectation. Nevertheless we obtain consistency with experiment only if the ration m_s/m_b is relatively large implying $m_s(1GeV) \geq 180MeV$.

It is remarkable that the simplest ansatz for the breaking of the "democratic symmetry", one which nature follows in the case of the pseudoscalar mesons, is able to reproduce the experimental data on the mixing between the second and third family. We interpret this as a hint that the eigenstates of the symmetry l_i, q_i respectively, and not the mass eigenstates, play a special rôle in the physics of flavour, a rôle which needs to be investigated further.

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